Toolkit 2:

Back-of-the-Envelope Calculations

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1. Introduction

Understanding the performance and impacts of energy systems requires an interdisciplinary approach that brings together scientific, technical, economic, social, political, and environmental opportunities and impacts of the energy system.

This set of chapters is organized into a series of Energy Toolkits that examine many of the fundamental skills that will be required to become an expert in the assessment, design, and critical appraisal of energy systems.

Energy Toolkit 2 focuses on back-of-the-envelope calculations and, simultaneously, introduces a number of energy-related concepts and topics that will be covered throughout the course. Thus, you should pay attention to both the method and the material. If the material is unfamiliar, do your best to follow the calculations. All of the topics introduced in this chapter will be explored in much greater detail in subsequent chapters.

Back-of-the-envelope calculations are a true art form, and one that, like throwing a knuckle-ball in baseball, you can teach the mechanical steps, but the synthesis takes a 'feel' for the methods. Over the course of this book, you will be working to develop your back-of-the-envelope calculation skills.

Back-of-the-envelope calculations can provide quick approximations, rough estimates, and a sense of the appropriate order of magnitude. Mechanically, back-of-the-envelope calculations take a simple and freewheeling form that draws upon unit conversions, careful attention to significant figures, an interdisciplinary feel for the key physical, economic, social laws and rules that you might expect a system to follow (such as pulling out of your memory or from a reference the energy of water in reservoir must be proportional to the [mass of the water] times [how far it falls] for a quick assessment of a proposed hydropower project), and the range of simple mathematics reviewed in Toolkit 1: Energy Units and Fundamentals of Quantitative Analysis.

The range of methods and applications of back-of-the-envelope techniques can be found in the plethora of books that employ them. Just the titles show you the delight the authors have in developing and using these methods:

> *Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin*
>
> *How Many Licks?: Or, How to Estimate Damn Near Anything*
>
> *Street-Fighting Mathematics: The Art of Educated Guessing and Opportunistic Problem Solving*
>
> *How to Measure Anything: Finding the Value of Intangibles in Business*

In this chapter we will develop, use, and critique back-of-the-envelope calculations. Above all, they should be clear and simple, as opposed to the more commonly stated but less useful moniker 'quick and dirty' because a good back-of-the-envelope calculation can and should not be dirty and may or may not be quick.

Mathematically, back-of-the-envelope should follow several simple rules.

First, they must be accurate in terms of the units. In fact, these calculations are often little more than a correct unit analysis that is then applied to a question. Many energy calculations, like many problems in physics, can be solved partially or completely by using a well-designed unit analysis.

Second, they must strictly adhere to the challenges of significant figures. Because these calculations are often no more than estimates based on orders of magnitude, being careful about the significant figures, or even powers of ten, is vital. Third, these calculations are not intended to be the 'last word' on a problem, but the first numeric assessment and feeling for the form of an answer. As a result, clarity of formulation is important, so that successively better numerical values can be inserted.

The art of the back-of-the-envelope calculation has been advanced and refined by scholars and practitioners alike. The physics Nobel Laureate and Harvard professor, Edward Purcell, demonstrated to many researchers just how perceptive back-of-the-envelope calculations could be. He argued, in fact, that you can often get the majority of the way to understanding an issue with a clear and incisive first analytic take on the issue.

More closely tied to environment and energy issues, my colleague and friend John Harte of UC Berkeley's Energy and Resources Group, has written two beautiful books – *Consider a Spherical Cow* and its sequel, *Consider a Cylindrical Cow* – that guide students and practitioners through a series of increasingly sophisticated back-of-the-envelope calculations focusing on environmental problems.

The following sections provide an introduction to back-of-the-envelope calculations and illustrate how they can be worked through. Importantly, back-of-the-envelope calculations can often be solved in multiple ways or require using reasonable (order of magnitude) estimates, so the solutions shown here will likely not be the *only* possible solution.

2. Defining a Metric for Electricity Efficiency and Savings: The Rosenfeld

This section has been adapted from Koomey, Jonathan*, et al.* (2010) "Defining a Standard Metric for Electricity Savings." *Environmental Research Letters* **5** 014017. *References have been removed for readability; see original article for citations and additional information.*

In the three decades since the energy crises of the 1970s we have learned a great deal about the potential for energy efficiency and the means to deliver it cost effectively and reliably. In the early days, many analysts still held to the now discredited idea of an "ironclad link" between energy use and economic activity, which implied that any reduction in energy use would make our society less wealthy.

A range of cross-country comparisons and assessments of technical capacity have since demonstrated that there are many ways to produce and consume goods and services, some energy efficient and others not. It is now clear the available efficiency resource is enormous, inexpensive, and largely untapped, making it an important option for reducing climate risks and improving energy security.

Perhaps most interesting, this resource of energy efficiency is not only available for rich communities already using large quantities of energy, but is a resource that even poor rural communities with access to no electricity or only a few kilowatt-hours can also exploit. This is an issue we will return to later on in the book.

All of this may sound great, and it can be, but energy efficiency also illustrates the problems of focusing on only one aspect of the energy equation. Energy efficiency gains are not simply technical issues of replacing an inefficient light bulb with a better one. They require both attention to the full system and arguably the most challenging issue of all, an approach that looks across multiple disciplines and methodological approaches. This has been the failure of so many well-meaning efforts, and one where education and this book is intended to arm the next generation of energy innovators.

The increased focus on energy efficiency for shaping our energy future highlights the need for simple tools to help understand and explain the size of the potential resource. One technique that is commonly used in that effort is to build on the idea first introduced by Amory Lovins over four decades ago of 'negawatts'. A negawatt is a watt of electricity generation not needed due to efficiency. Today there is even discussion of 'virtual power plants' where efficiency savings are aggregated and traded. The challenge remains – visualizing these savings and making the technical, economic, and policy case for their value. In this section we will do just that.

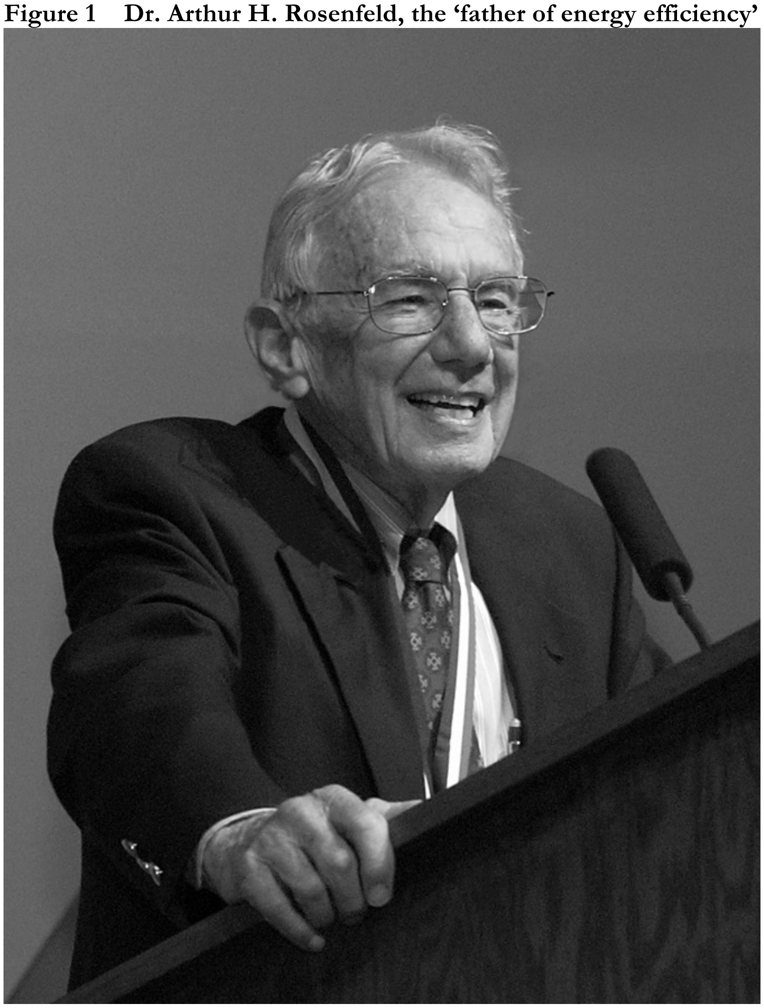
2.1. Dr. Art Rosenfeld – "Father of Energy Efficiency"

Dr. Rosenfeld, see Figure 1, received his PhD in physics under the supervision of Enrico Fermi, creator of the first sustained fission reactor. Fermi directed the wartime effort that created Chicago Pile-1 (CP-1), the world's first nuclear reactor. CP-1 was built on a racketball court, under the abandoned west stands of the original Alonzo Stagg Field stadium, at the University of Chicago. The first self-sustaining nuclear chain reaction was initiated in CP-1 on December 2, 1942. The site was designated a National Historic Landmark in 1965.

This work was, in fact, directed as a collaboration between Fermi and Leó Szilárd, discoverer of the chain reaction. It contained a critical mass of fissile material, together with control rods, and was built as a part of the Manhattan Project by the University of Chicago Metallurgical Laboratory. The shape of the pile was intended to be roughly spherical, but as work proceeded Fermi began a famous back of the envelope calculation where he determined that critical mass could be achieved without finishing the entire pile as planned. This was very fortunate, because building a spherical pile of bricks is somewhat more difficult. The final calculation evolved into something far more complex than a basic back of the envelope effort, but was in final form a mixture of geometry and an assessment of the density of neutrons that came from the pile. We will return to this calculation in the nuclear toolkit chapter.

Fermi, referred to by some as the 'father of atomic energy,' was a refugee and professor at the University of Chicago. Art Rosenfeld made his transition from particle physics to studying energy efficiency at the time of the first oil embargo. Over the past 35 years Art has been at the forefront of efforts to improve the efficiency of energy use around the world and has devoted special care to making the results of complex energy analysis understandable to a lay audience. Art has come to be known, to some, as the 'father of energy efficiency.'

Figure 1 Dr. Arthur H. Rosenfeld, the 'father of energy efficiency'



2.2. Standard Avoided Power Plant

For years, Dr. Rosenfeld has characterized oil savings in terms of "Arctic Refuges saved" and electricity savings in terms of "avoided power plants" to emphasize that supply and demand side policy options are fungible and that replacing power plants with more efficient energy technologies would be beneficial for consumers' energy bills and for the environment.

In lectures and calculations, Dr. Rosenfeld worked with a standard avoided power plant, a 500 MW coal power plant operating 5,000 hours per year. In 2010, a group of more than 50 of Dr. Rosenfeld's colleagues proposed that a new unit, the Rosenfeld, be created in his honor. The Rosenfeld would be a measure of energy *savings* – electricity use that would be avoided because demand has been decreased through conservation or the use of energy-efficient technologies.

The standard avoided plant used by Dr. Rosenfeld inspired the selection of characteristics for determining the value of the Rosenfeld, see Table 1. The following sections briefly introduce each of these characteristics – these are concepts that we will cover much more extensively later in Toolkit 4, which delves more deeply into power plants.

Table 1 Assumptions Used to Determine the Rosenfeld

Fuel choice	Coal
Capacity	500 MW
Capacity factor	70%
Efficiency	33%
System losses	7%

2.2.1. Fuel Choice

Since at least the 1950s, roughly half of all electricity generated in the United States has come from coal-fired power plants. In 2010, 45% of electricity came from coal, 24% from natural gas, 20% from nuclear power, 10% from renewable energy sources, and 1% from petroleum. Coal-fired power plants were also responsible for about 80% of domestic carbon dioxide (CO_2) emissions from the electric power sector and 35% of emissions from all energy consumption.

Electricity generation is the primary use of coal. Coal is combusted to produce steam, which turns turbines to produce electricity. The details of this process will be explored in Toolkit 4. Some coal plants produce not only electricity, but also useful thermal energy, such as heat or steam. These plants are referred to as cogenerating plants.

Between 2000 and 2007, 151 new coal-fired power plants were proposed in the United States; 10 have been completed, 25 more are under construction, and 59 have been canceled or indefinitely deferred. In 2007, coal plants accounted for more than 30% of total installed capacity in the United States, or more than 300 GW out of almost 1,000 GW of total capacity.

Coal plants are also ubiquitous in other countries and are responsible for a substantial percentage of global CO_2 emissions. For example, Section 3 of this Toolkit looks at China's coal-fired power plants and associated CO_2 emissions.

2.2.2. Capacity

The term *capacity* refers to a power plant's maximum rate of electricity generation. Capacity is typically measured in megawatts (MW). Power plants vary greatly in their capacity, ranging from just a few megawatts to a few thousand megawatts. *Nameplate* or *nominal capacity* is the maximum power output that a plant can generate. *Net capacity* refers to the rate of electricity output available for use after the power needed to run the plant has been subtracted out.

The U.S. Energy Information Administration (EIA) provides a wealth of data about energy generation and consumption in the United States, including information on power plants, fuels, and costs. These data will regularly be drawn upon in this course and you may find it useful to familiarize yourself with their website and the kinds of data it contains.

EIA's *Electric Power Annual 2007* and *2009* show that total capacity for U.S. coal fired power generation was quite stable over the period 1996 to 2009, starting and ending at just over 300 GW, see Table 2.

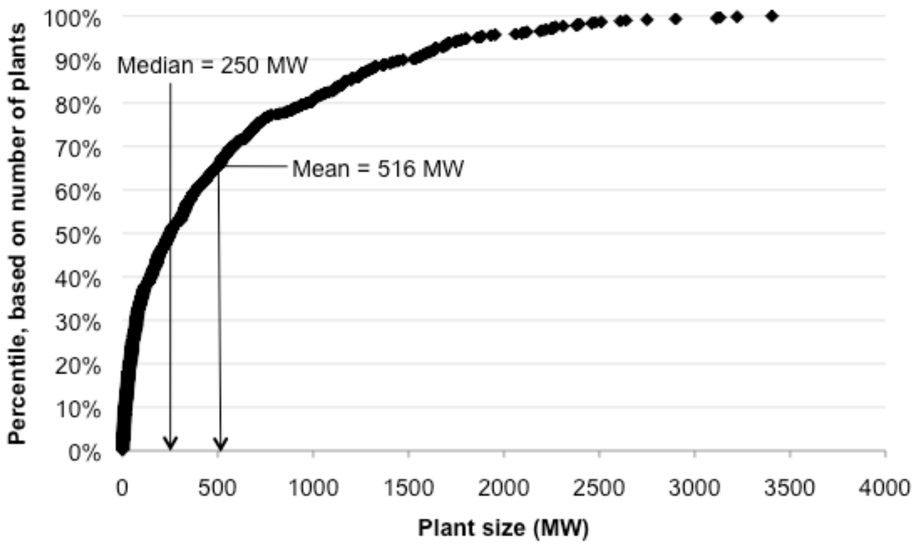
Table 2 Characteristics of Existing U.S. Coal-fired Power Plants

	Coal fired capacity GW	**Net generation** TWh	**Capacity factor** %	**Coal consumed** 10^6 short tons	**Heat content of utility coal** MBtu/short ton	**Average HHV efficiency** %
1996	313	1,785	65.2	907	20.55	32.9
1997	314	1,845	67.2	932	20.52	32.9
1998	316	1,874	67.7	946	20.52	32.9
1999	315	1,881	68.1	950	20.49	33.0
2000	315	1,966	71.0	995	20.51	32.9
2001	314	1,904	69.2	973	20.34	32.8
2002	315	1,933	70.0	988	20.24	33.0
2003	313	1,974	72.0	1,014	20.08	33.1
2004	313	1,978	71.9	1,021	19.98	33.1
2005	313	2,013	73.3	1,041	19.99	33.0
2006	313	1,991	72.6	1,031	19.93	33.1
2007	313	2,016	73.6	1,047	19.91	33.0
2008	313	1,986	72.2	1,042	19.71	33.0
2009	314	1,756	63.8	935	19.54	32.8
Average			**69.8**			**33.0**

Notes: Coal consumed, capacity, and net generation include all coal-fired power plants in the U.S., including utility and non-utility central station plants as well as industrial cogeneration plants. Coal fired capacity, net generation, and coal consumed taken from US DOE *Electric Power Annual 2007* through 2007 and US DOE *Electric Power Annual 2009* for 2008 and 2009. Heat content of coal taken from Table A-5 in US DOE Annual Energy Review 2007 through 2007 and from US DOE *Annual Energy Review 2009* for 2008 and 2009. MBtu = million Btus. Capacity factor calculated from capacity and net generation assuming 8760 hours for non-leap years and 8784 hours for leap years. Power plant efficiency (higher heating value, or HHV) calculated by converting net generation to Btus assuming 3412 Btus/kWh and then dividing by the product of coal consumed and heat content of utility coal.

In 2007, the median nameplate capacity for existing non-cogenerating U.S. coal plants was 250 MW. The mean nameplate capacity was about 500 MW. Coal plants ranged in capacity from less than 1 MW to about 3500 MW. Most of the smaller plants (those less than 200 MW) tend to be from the 1960s or earlier, while the larger plants tend to be newer, from 1970s or later. See Figure 2 for the distribution of capacity for existing U.S. coal-fired power plants.

Figure 2 Cumulative Distribution of Capacity for Existing U.S. Coal-fired Power Plants in 2007.



Source: U.S. Energy Information Administration.

2.2.3. Capacity Factor

Capacity factor refers to the ratio of the actual amount of electricity generated per year to the maximum amount of energy that could have been generated if electricity had been produced continuously at the rate of the nameplate capacity. Capacity factor is defined in Equation 1:

$$\text{Capacity factor} = \frac{\text{Actual generation (kWh)/year}}{\text{Maximum generation (kWh)/year}} \tag{1}$$

Dividing numerator and denominator by the number of hours per year (8766 hours when averaged across leap and non-leap years) we get a second formulation of capacity factor:

$$\text{Capacity factor} = \frac{\text{Average output capacity (MW)}}{\text{Nameplate capacity (MW)}} \tag{2}$$

New coal plants typically have high capacity factors (up to 90%), but the average capacity factor for all existing U.S. coal plants has hovered around 70% for over a decade, see Table 2.

2.2.4. Power Plant Efficiency

Power plant efficiency refers to the ratio of energy output (electricity) to energy input (fuel) of the power plant.

$$\text{Power plant efficiency } (\eta) = \frac{\text{energy output}}{\text{energy input}} \tag{3}$$

For coal plants, power plant efficiency can be more specifically written as:

$$\text{Power plant efficiency } (\eta) = \frac{\text{energy content of electricity generated } (MJ_e)}{\text{energy content of fuel source } (MJ_{th})} \tag{4}$$

Equation 4 introduces a useful convention for differentiating between the thermal energy produced when coal is combusted and the electric energy generated by the power plant. Thermal energy is indicated with a subscript (th) and electricity is indicated with a subscript (e).

When coal is combusted, it produces thermal energy or heat. Every fuel produces a certain amount of heat per unit of fuel burned, a measurement known as its heating value. Different types of coal have different heating values, ranging roughly between 25 and 30 MJ/kg; see the Appendix for a more detailed table of fuel heating values. During combustion, water is one of the products. The amount of heat produced when liquid water is produced is called the *higher heating value* (HHV); the amount of heat produced when steam is produced is called the *lower heating value* (LHV). The efficiency with which a power plant is able to convert fuel into electricity is typically given in terms of its higher heating value efficiency.

In recent decades, existing coal steam power plants convert coal to energy with an efficiency of about 30 to 40%, depending on age, pollution control, and technology type. Table 2 shows that between 1996 and 2009, the average efficiency of existing coal steam plants have steadily had a higher heating value efficiency of 33%.

2.2.5. System losses

Table 3 shows data from EIA's *Electric Power Annual 2007* on the supply and disposition of electricity in the U.S. from 1995 to 2007. Losses are expressed as a percentage of the sum of electricity sales, direct use by power plants, and exports. These losses range from 5.7% to 7.4% with a simple average of 6.8% over that period.

Table 3 U.S. Average Transmission and Distribution (T&D) Losses Over Time

	Total electric industry sales	**Direct use**	**Total exports**	**Losses and unaccounted for**	**T&D losses**
	TWh	TWh	TWh	TWh	%
1996	3,101	153	3	231	7.1
1997	3,146	156	9	224	6.8
1998	3,264	161	14	221	6.4
1999	3,312	172	14	240	6.9
2000	3,421	171	15	244	6.8
2001	3,394	163	16	202	5.7
2002	3,465	166	16	248	6.8
2003	3,494	168	24	228	6.2
2004	3,547	168	23	266	7.1
2005	3,661	150	20	269	7.0
2006	3,670	147	24	266	6.9
2007	3,754	159	20	264	6.7
2008	3,733	132	24	287	7.4
2009	3,597	127	18	261	7.0
Average					**6.8**

Notes: Data on electric industry sales, direct use, exports, and losses are taken from US DOE *Electric Power Annual 2007* through 2007 and US DOE *Electric Power Annual 2009* for 2008 and 2009. T&D losses calculated as a percentage of sales plus direct use plus exports.

2.3. Defining the Rosenfeld

So, let's do the math. The Rosenfeld is equal to the avoided annual electricity production of a 500 MW coal power plant that, in summary, is 33% efficient, loses 7% of the electricity produced in transmission and distribution to end-users, and has a capacity factor of 70%. With this information, we can calculate the Rosenfeld as follows:

$$\begin{aligned} 1\text{ Rosenfeld} &= [\text{Capacity}][\text{Capacity Factor}][1 - \text{System Losses}] \\ &= 500MW_e \left[\frac{1000kW}{1MW}\right]\left[\frac{8760hr}{1yr}\right][0.70][1 - 0.07] \\ &= 3\times 10^9 kWh/yr \end{aligned} \tag{5}$$

This combination of parameters yields annual electricity delivered at the meter of about 3 billion kilowatt-hours per year (3×10^9 kWh/yr). In other words, one Rosenfeld is equal to *avoiding* the production of 3 billion kilowatt-hours of *delivered* electricity per year from an average U.S. coal power plant. This calculation serves as a definition for a suitable metric to be known as *the Rosenfeld*. It is also based around round and easy to remember numbers – a hallmark of a back-of-the-envelope calculation.

A few more calculations help put the Rosenfeld into context. The Rosenfeld is given as a measure of power, so let's do a few conversions to get a sense of the amount of energy equivalent to a Rosenfeld-year. Since the Rosenfeld measures the amount of *delivered* electricity, begin by calculating the amount of delivered energy equivalent to one Rosenfeld-year in exajoules (EJ):

$$\begin{aligned} \text{Delivered Energy} &= \left[3\times10^9 \frac{kWh}{yr}\right]\left[\frac{3.6\times10^6 J}{1kWh}\right]\left[\frac{1EJ}{10^{18}J}\right][1\text{ year}] \\ &= 0.0108\, EJ \\ &= 1\times10^{-2} EJ \end{aligned} \tag{6}$$

From this, we can calculate the amount of primary energy, the amount of energy embodied in the coal, in one Rosenfeld-year by taking into account the average plant efficiency:

$$\begin{aligned} \text{Primary Energy} &= [0.0108\, EJ]\left[\frac{1}{0.33}\right] \\ &= 0.03\overline{27}\, EJ \\ &= 3\times10^{-2}\, EJ \end{aligned} \tag{7}$$

In sum, there are 100 Rosenfeld-years per exajoule of delivered energy or about 30 Rosenfeld-years per exajoule of primary energy.

The Rosenfeld can also be thought of in terms of reductions in CO_2 emissions. Coal is a carbon-intensive fuel, even relative to other fossil fuels, see Table 4. The generation of each kilowatt-hour of electricity from coal releases about 1 kilogram of CO_2 into the atmosphere. Thus, one Rosenfeld can also be thought of as avoiding the emission of almost exactly 3 million metric tons of CO_2 (MMT CO_2) per year:

$$1\ \text{Rosenfeld} = \left[3\times10^9\ \frac{kWh}{yr}\right]\left[\frac{1kg(CO_2)}{1kWh}\right]\left[\frac{1tonnes}{10^3 kg}\right]\left[\frac{1MMT}{10^6 tonnes}\right] = 3MMT(CO_2)/yr \tag{8}$$

Table 4 Carbon emission factors for electricity delivered to the meter

Fuel	Efficiency HHV	Emissions factor gC/kWh fuel	Emissions factor gC/kWh electricity delivered	Index Existing coal = 1.0	Notes
Existing Plants:					
Steam turbine Coal	33.0	88.1	286	1.00	1, 2
Steam turbine Natural gas	32.6	49.4	162	0.57	1, 3
Gas turbine Natural gas	29.8	49.4	177	0.62	1, 3
Combined cycle Natural gas	45.8	49.4	115	0.4	1, 3
New Plants:					
Steam turbine, scrubbed coal	37.1	88.1	254	0.89	1, 4
Advanced combined cycle Natural gas	50.5	49.4	105	0.37	1, 4

Notes:

1. Complete list of missions factors for fossil fuels and sources can be found in original article.
2. Steam turbine efficiency for average existing US coal plants from 1996-2009 taken from Table 2.
3. Steam turbine, gas turbine, and combined cycle efficiencies for existing oil and gas plants calculated from higher heating value (HHV) heat rates in the Electric Power Annual 2007, Table A7, which represent an average for existing plants in 2007. The Electric Power Annual table does not differentiate between residual oil and distillate oil steam turbine efficiencies so we assume these are the same.
4. Efficiencies for 2008 new plants derived from heat rates in Assumptions to the AEO 2009, Table 8.2.

The Rosenfeld assumes a number of simplifications that help make quick calculations and cogent interpretation of results from studies of energy efficiency, see Table 5 for a thorough summary of these assumptions. To use the Rosenfeld, analysts have to remember the numbers associated with the power plant characteristics (500 MW, 70% capacity factor, 7% T&D losses, 33% HHV efficiency), and the number 3 (which evokes 3 billion kWh per year [3×10^9 kWh/yr] saved at the meter, 3 million metric tons of carbon dioxide avoided per year [3×10^6 MMT CO_2], and 30 Rosenfeld-years per exajoule of primary energy).

Six hundred billion kilowatt-hours per year is the equivalent of about 200 Rosenfelds, or about 200 typical coal-fired power plants, which together emit 600 million metric tons of CO_2 per year. This simple calculation adds real physical meaning to the electricity savings.

The Rosenfeld can best be used in rough back-of-the-envelope calculations and high-level summaries of analysis results for less technical audiences. If an efficiency technology or policy would save 3 billion kilowatt-hours per year at the meter, it saves one Rosenfeld, or one 500 MW coal plant operating at 70% capacity factor in that year (assuming 7% T&D losses). It also saves 3 million metric tons of CO_2 per year (assuming all the savings come from conventional coal plants). These parameters satisfy the initial criteria of simplicity of presentation, ease of recall, intuitive plausibility, physical meaning, and policy relevance.

Table 5 Estimating electricity delivered and carbon emitted from a typical coal plant in the United States

	Units	Value	Notes
Electricity generated			
Capacity	MW	500	1
Capacity factor	%	70%	2
Hours per year	hours	8766	3
Assumed T&D losses	%	7%	4
Total electricity generated at the busbar	Billion kWh/year	3.07	5
Total electricity delivered to the meter	Billion kWh/year	2.87	6
Site energy (HHV)	Quadrillion Btus/year	0.010	7
	Exajoules/year	0.010	8
Primary energy (HHV)	Quadrillion Btus/year	0.032	9
	Exajoules/year	0.034	8
Carbon emitted			
Coal carbon burden (based on HHV)	gC/kWh fuel	88.1	10
Efficiency (based on HHV)	%	3%	11
Carbon burden at the busbar	gC/kWh electricity generated	267	12
Carbon burden at the meter	gC/kWh electricity generated	286	13
Carbon emissions	Million metric tons C/yr	0.82	14
	Million metric tons CO_2/yr	3.01	15

Notes:

1. Capacity is based on average existing US coal plants from EIA-860 survey results as summarized in Figure 2.
2. Capacity factor is the average for existing US coal plants from 1996 to 2009, see Table 2.
3. Hours per year is an average over leap years and non-leap years.
4. T&D (transmission and distribution) losses are typical for the US utility system (see Table 3), rounded up to 7% for ease of recall.
5. Total electricity generated at the busbar is the product of capacity, capacity factor, and hours per year, expressed using the American notation of billion equating 109.
6. Total electricity delivered to the meter is total electricity generated divided by (1+percentage T&D losses).
7. Site energy in quadrillion Btus/year calculated by multiplying kWh per Rosenfeld at the meter by 3412 Btus/kWh.
8. Quadrillion btus converted to exajoules using the factor 1055.1 joules/Btu.
9. Primary energy in quadrillion Btu.year calculated by converting the efficiency described in footnote 11 to a heat rate (primary energy per kWh), then multiplying that heart rate times (1+percentage T&D losses) and multiplying again by the number of kWh per Rosenfeld.
10. The carbon burden of coal is expressed in grams of carbon (C) per kWh of fuel (fuel converted to kWh assuming 3412 Btus/kWh). This carbon burden is taken from EIA for 2006.
11. Power plant efficiency, in higher heating value (HHV) terms, is the average for existing US coal plants from 1996 to 2009 from Table 2.
12. Carbon burden at the busbar (calculated in grams of carbon per kWh generated) is calculated as the ratio of the coal C burden, see original paper.
13. Carbon burden at the meter is the carbon burden at the busbar times (1+percentage T&D losses).
14. Carbon emissions in million metric tons are the product of electricity consumed at the meter and the carbon burden at the meter.
15. Carbon dioxide emissions in million metric tonnes are equal to carbon emissions times the ratio of the molecular weights of carbon dioxide (44) and carbon (12).

2.4. Using the Rosenfeld

Now let's work through a few problems that use the Rosenfeld. Table 6 provides a summary of equivalent values for the Rosenfeld, which may be useful for working through these problems.

Table 6 Summary of Rosenfeld equivalences

1 Rosenfeld is equivalent to…	
the electricity of 1 avoided coal-fired power plant	$3{\times}10^9$ kWh/yr
the CO_2 emissions of 1 avoided coal-fired power plant	3 MMT CO_2/yr
1 Rosenfeld-year is equivalent to…	
in terms of primary energy	$1{\times}10^{-2}$ EJ
in terms of delivered energy	$3{\times}10^{-2}$ EJ

Example 1 If enough electricity is saved to avoid burning 100g of coal, how many Rosenfeld-years of savings does that represent?

Solving this problem requires a more advanced unit analysis. The solution breaks the unit analysis into several steps here, but could also be written as a single, long equation.

The first step is to calculate how much thermal energy (MJ_{th}) 100g of coal can produce.

$$\text{Thermal energy} = [100g]\left[\frac{1kg}{1000g}\right]\left[\frac{29.3MJ_{th}}{1kg\ coal}\right]$$

$$= 2.93\ MJ_{th}$$

Now calculate how much of the thermal energy is converted to electricity (MJ_e). This step uses the average power plant efficiency included in the assumptions about the Rosenfeld.

$$\text{Electricity} = 2.93MJ_{th}\left[\frac{0.33MJ_e}{1MJ_{th}}\right]\left[\frac{1kWh}{3.6MJ_e}\right]$$

$$= 2.69kWh$$

Keep in mind that this value represents the amount of electricity produced at the power plant. Since the Rosenfeld is calculated based on delivered electricity, another step is required to take into account system losses before calculating the number of Rosenfeld-years saved.

$$\text{Rosenfeld-year} = 2.69kWh[1-0.07]\left[\frac{1\ Rosenfeld}{3{\times}10^9\ kWh/yr}\right]$$

$$= 8.33{\times}10^{-11}\text{Rosenfeld-years}$$

$$= 8{\times}10^{-11}\text{Rosenfeld-years}$$

Example 2 **If your refrigerator uses a constant 56 W of electricity, but you are planning on replacing it with a new, more efficient unit. What would the wattage of the replacement fridge have to be in order to see energy savings of 67 nanoRosenfelds over the course of a year?**

The set-up for this problem combines unit analysis with some basic algebra.

$$67 nanoRosenfelds = [56W - x]\left[\frac{1kW}{10^3 W}\right]\left[\frac{8760hr}{1yr}\right]\left[\frac{1Rosenfeld}{3\times10^9\, kWh/yr}\right]\left[\frac{10^9 nanoRosenfeld}{1Rosenfeld}\right]$$

$$67 nanoRosenfelds = [56W - x]\left[\frac{8760\times10^9 nanoRosenfelds}{3\times10^{12} W}\right]$$

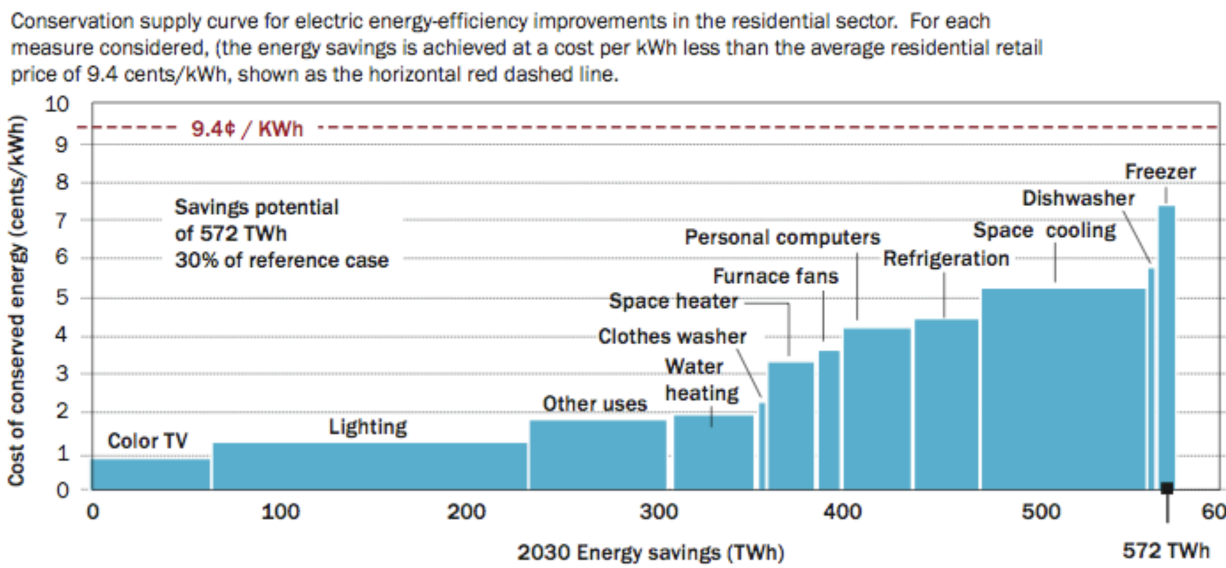
$$56W - x = 22.95W$$

$$x = 33.05W$$

$$x = 30W$$

Example 3 **Figure 3, shows the potential U.S. residential sector efficiency savings of almost 600 billion kWh/year in 2030. What does that number mean in terms of power plants avoided?**

Figure 3 Potential for residential electric savings for 2030.



Source: APS, Energy Future: Think Efficiency, 2008.

The set-up for this problem involves a straightforward unit analysis that requires converting electricity savings to power plants avoided. Before attempting the conversion, however, it is important to understand how to read the figure, which shows the potential electricity savings from the U.S. residential sector on a conservation supply curve reproduced from the recent authoritative study on energy efficiency by the American Physical Society, *Energy Future: Think Efficiency* (APS, 2008). Conservation supply curves such as this one provide technical-economic estimates of potential energy savings associated with various technological improvements, in this case electricity savings from the residential sector.

In Figure 3, the x-axis shows the amount of potential electricity savings in 2030 and the y-axis the cost per kilowatt-hour for implementing the technology. Each bar represents a different class of energy-efficiency improvement, with the width showing the electricity savings associated with implementation and the height representing the cost per kilowatt-hour of the improvement. In Toolkit 5, we will dig deeper into how these cost of conserved energy calculations are done, but for now it is only necessary to understand how to interpret the graph.

In this case, we can see that the greatest amount of electricity savings come from lighting improvements and that these savings can be accomplished at a very low cost, just over 1 cent/kWh. Energy efficient upgrades to dishwashers and clothes washers, on the other hand, both provide only modest electricity savings at about 6 cents/kWh and 2 cents/kWh, respectively. Notably, all of the improvements shown on this graph cost less per kilowatt-hour than the average per kilowatt-hour cost of electricity, indicated by the dotted red line.

With that in mind, let's finally calculate how many average power plants could be avoided if the residential electricity savings identified in the conservation supply curve were realized.

$$\text{Residential electric savings} = [600\times10^{9}\, kWh/yr]\left[\frac{1\ average\ power\ plant}{3\times10^{9}\, kWh/yr}\right]$$

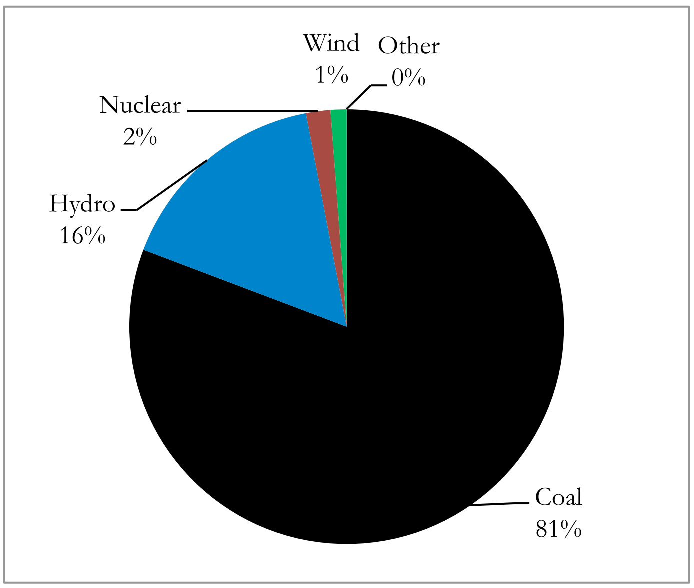
$$= 200 \text{ average power plants}$$

3. China's Coal-fired Power Efficiency and Carbon Emissions

China is the world's fastest growing user of fossil fuels and the nation with the greatest annual greenhouse gas emissions. China relies on coal for 70% of its primary energy supply and about 80% of its electricity generation. As such, China, especially China's coal-fired power sector, simultaneously presents the most challenging and critical test for energy efficiency and carbon emission control. In this back-of-the-envelope calculation, we will explore what the impact would be if China's power sector upgraded to the world's most efficient technologies on global energy use and global greenhouse gas emissions.

By the end of 2010, China's total power generation was 4,228 TWh (about 20% of global total), of which 3,414 TWh (80.7%) was from coal. Hydropower, nuclear power, and renewables filled out the rest of China's electricity portfolio, respectively accounting for 16.2%, 1.8%, and 1.2% of generation, see Figure 4.

Figure 4 China's Electricity Mix of 2010



Source: *Electricity Quick Statistics 2010*, China Electricity Council. 2011.

Now, let's work through a series of back-of-the-envelope calculations to understand the development and current status of the Chinese coal power sector. We will also explore why China is important to global energy and climate policy.

3.1. Efficiency of China's Coal-fired Power Plants

The average coal-fired power plant in the United States has an efficiency of 33%. Official statistics from China do not report power plant efficiency directly. Instead, the China Electricity Council reports the amount of coal, or equivalent, consumed to generate 1 kilowatt-hour of electricity as a proxy for power plant efficiency. The units for these proxy measurements are given in terms of *grams of coal equivalent* (gce) per kilowatt-hour. The use of "coal equivalent" allows the inclusion of not just coal, but also natural gas, oil, and other fuels, where 1 kilogram of coal-equivalent equals 7,000 kilocalories (kcal).

Example 4 **According to the China Electricity Council, the average coal consumption of the coal power supply in 2010 was 335gce/kWh. What is the average efficiency of China's coal-fired power plant?**

Solving this problem involves a straightforward conversion, but the set-up requires a little attention to make sure that the denominator represents the energy content of the fuel source and the numerator represents the energy content of the electricity generated.

$$\text{power plant efficiency} = \frac{\text{energy content of electricity generated}}{\text{energy content of fuel source}}$$

$$= \left[\frac{1kWh}{335gce}\right]\left[\frac{10^3gce}{1kgce}\right]\left[\frac{1kgce}{7\times10^3kcal}\right]\left[\frac{1kcal}{10^3cal}\right]\left[\frac{1cal}{4.1868J}\right]\left[\frac{3.6\times10^6J}{1kWh}\right]$$

$$= 36.667\%$$

$$= 36.7\%$$

From this, we can conclude that the average efficiency of China's coal-fired power plants is higher than the average efficiency of the United States' coal-fired power plants.

3.2. Coal Consumption in China

We are now able to calculate how much coal is being consumed to power China.

Example 5 **Recall that in 2010, China generated 3,414 TWh of electricity from coal-fired power plants. If 1 tonne of raw coal is equal to 0.7143 tonnes of coal-equivalent (tce), how much raw coal was consumed in 2010 to generate electricity?**

$$\text{Raw coal consumed} = 3{,}414TWh\left[\frac{335gce}{1kWh}\right]\left[\frac{10^9kWh}{1TWh}\right]\left[\frac{1tce}{10^6gce}\right]\left[\frac{1t_{raw\ coal}}{0.7143tce}\right]$$

$$= 1{,}601{,}133{,}977 \text{ tonnes of raw coal}$$

$$= 1.60\times10^9 \text{ tonnes of raw coal}$$

Example 6 **According to the Chinese Coal Transportation and Distribution Association, China produced 3.3×10⁹ tonnes of raw coal in 2010. What portion of this coal was burned to generate electricity to meet China's electricity needs?**

$$\text{Percentage of coal consumed for electricity} = \left[\frac{1.601133977\times10^9t_{raw\ coal}}{3.3\times10^9t_{raw\ coal}}\right][100]$$

$$= 48.52\%$$

$$= 50\%$$

Wow, that's a lot!

Over the decade from 2001 to 2010, China has reduced its coal consumption by 5.7 gce/kWh. Meanwhile, China's electricity generation has grown at an average rate of 11.9% per year.

Example 7 What is the total amount of raw coal that China will consume during the decade 2011 to 2020? Assume that China's rates of coal consumption and electricity production continue to decrease and increase, respectively, at the same rates as the previous decade.

The set-up for this calculation is very similar to Example 5 for each of the ten years and then summing the annual totals. Spreadsheets can be very useful for solving problems like this. The set-up and solution for this problem can be found in the spreadsheet for Toolkit 2, available online through bspace.

$$\text{Total raw coal consumed} = \sum_{i=2011}^{2020} \mathrm{M}_i$$
$$= 2.79\times10^{10}\ tonnes$$

3.3. Impacts of Increasing Power Plant Efficiency in China

Although the average efficiency of China's coal-fired power plants has increased over the past decade, there remains room for improvement.

Example 8 If we assume China's power plants were all upgraded to global best practice efficiency at 45% in 2011, how much raw coal would this upgrade save through 2020?

The set-up for this problem begins with some algebra and unit conversions to calculate the grams of coal equivalent consumed to produce 1 kilowatt-hour of electricity from a coal-fired power plant with 45% efficiency.

$$x\ \text{gce} = \frac{1kWh}{0.45}$$
$$= \left[\frac{1kWh}{0.45}\right]\left[\frac{3.6\times10^6 J}{1kWh}\right]\left[\frac{1cal}{4.1868J}\right]\left[\frac{1kcal}{10^3 cal}\right]\left[\frac{1kgce}{7\times10^3 kcal}\right]\left[\frac{10^3 gce}{1kgce}\right]$$
$$= 273\ \text{gce}$$

Now, we can modify our spreadsheet design from Example 7 to solve for the amount of coal used annually if we assume a fixed value of 273 gce as the average coal consumption per kilowatt-hour from 2011 through 2020, see the worksheet for Example 8.

$$\text{Total raw coal consumed}_{\text{high efficiency}} = \sum_{j=2011}^{2020} M_j = 2.55\times10^{10} tonnes$$

We can then estimate the savings from a high-efficiency scenario by subtracting this amount from the amount calculated in Example 7.

$$\begin{aligned}\text{Raw coal saved} &= 2.79\times10^{10} tonnes - 2.55\times10^{10} tonnes \\ &= 0.238\times10^{10} tonnes \\ &= 2.38\times10^{9} tonnes\end{aligned}$$

That's more than the amount of coal consumed in 2010!

4. BUILDING A BASIC ENERGY BUDGET

Thus far, our back-of-the-envelope calculations have largely focused on generating insights about country-level energy production and use in the United States and China, respectively. For a more complete understanding of the global energy landscape, it will be necessary to zoom out to compare energy data across countries, as well as to zoom in to explore the contours of energy data within countries. In this section, we will do a little of both in order to build a basic energy budget.

Consider the data displayed in Table 7, which includes selected energy statistics at the global, regional, and country levels. Total and per capita electricity consumption and carbon dioxide (CO_2) emissions are shown for all three scales. Each level of data provides some information about which certain generalizations can be made, but the data – and any calculations that employ them – also have their limits. What shortcomings do you see in the way the data are presented in Table 7?

Table 7 Total and per capita electricity and carbon dioxide (CO_2) emissions, 2009

Region / Economy	Population	Electricity consumption	CO_2 emissions	Per capita electricity consumption	Per capita CO_2 emissions
	million	TWh[a]	MT (CO_2)[b]	kWh/person	t(CO_2)/person
World	6,761	18,456	28,999	2,730	4.29
OECD	1,225	9,813	12,045	8,012	9.83
Middle East	195	638	1,509	3,278	7.76
Non-OECD Europe & Eurasia	335	1,407	2,497	4,200	7.46
Asia	2,208	1,637	3,153	741	1.43
Latin America	451	850	975	1,884	2.16
Africa	1,009	566	928	561	0.92
United States	307	3,962	5,195	12,884	16.90
Brazil	194	426	338	2,201	1.74
Canada	34	522	521	15,467	15.43
People's Republic of China	1,331	3,503	6,832	2,631	5.13
Ethiopia	83	4	7	45	0.09
India	1,155	690	1,586	597	1.37
Mexico	107	218	400	2,026	3.72

Notes: [a] Electricity consumption includes gross production and imports minus exports and losses.
[b] CO_2 emissions from fuel combustion only. Emissions are calculated using the IEA's energy balances and the *Revised 1996 IPCC Guidelines*.

Source: Table adapted from International Energy Agency (2011). *2011 Key World Energy Statistics*. Paris: International Energy Agency.

In the context of energy and society, data tables that provide total and per capita statistics are quite common, and the information they contain can be used to produce a variety of interesting and useful insights, as we have already seen in sections 2 and 3. However, these kinds of data often provide little or no information about how energy use is distributed within the population.

The data in Table 7 only provide part of the story. Questions to consider include:

- What percentage of the population has access to electricity, but cannot afford the cost to meet their basic energy needs?

- What percentage of the population lacks access to electricity most or all of the time?
- How do households that lack access to electricity meet their energy needs?

Electricity is just one kind of energy that people may have access to, but many do not. Indeed, about half of the world's population burns coal, wood, dung, and other fuels to heat their homes and cook their food. For many people, meeting basic energy needs is a time-consuming task with significant social consequences, including a heavy toll on public health from indoor combustion of coal and biomass fuels. These issues will be considered more in depth later in the course, but for now let's consider what a 'basic' budget of energy services looks like.

Conceptually, a basic energy budget consists of the various services that energy can provide:

$$\text{Energy needs} = (\text{heating \& cooking}) + (\text{lighting}) + (\text{other, } \textit{e.g.}\text{, computer, phone, etc.}) \quad (9)$$

An energy budget can then be constructed in a top-down manner, starting with a total amount and estimating the best way to apportion it for different uses, or in a bottom-up manner, identifying the minimum amount of energy required for each category and adding these together.

Example 9 What does a bare-bones energy budget look like? Give your answer in kilowatt-hours, even for non-electrical energy.

Begin with observed estimates of per person energy use for heating and cooking in several regions:

$$\text{Heating \& cooking} = \frac{1\ tonne(wood)}{year} \quad \text{(East Africa)}$$

$$= \frac{1\ GJ_{delivered}}{year} \quad \text{(MINES, India)}$$

$$= \frac{\{5 \text{ to } 8\}\ GJ_{delivered}}{year} \quad \text{(Mexico)}$$

The energy from combusting 1 tonne of wood can be converted into gigajoules using an estimate of the energy content of wood (20 GJ/tonne) and an estimate of cookstove efficiency (5%):

$$\text{Heating \& cooking (East Africa)} = \frac{1\ tonne(wood)}{year}\left[\frac{20\ GJ_{th}}{tonne(wood)}\right]\left[\frac{0.05\ GJ_{delivered}}{1\ GJ_{th}}\right]$$

$$= 1\ GJ_{delivered}/yr$$

Thus, the minimum amount of energy needed for heating and cooking ranges from 1 to 8 GJ of usable energy. This range can then be easily converted into kilowatt-hours:

$$\text{Heating \& cooking (low)} = 1\ \frac{GJ}{yr}\left[\frac{10^9 J}{1GJ}\right]\left[\frac{1kWh}{3.6\times10^6 J}\right]$$

$$= 278\ kWh/yr$$

$$= 300\ kWh/yr$$

$$\text{Heating \& cooking (high)} = 8\ \frac{GJ}{yr}\left[\frac{10^9 J}{1GJ}\right]\left[\frac{1kWh}{3.6\times10^6 J}\right]$$
$$= 2{,}222\ kWh/yr$$
$$= 2{,}000\ kWh/yr$$

Annual heating and cooking needs, thus, range from 300-2,000 kilowatt-hours for a basic energy budget in the regions considered; other locations (*e.g.*, Mongolia) might have greater heating needs. Minimum daily lighting needs can be estimated as one 60-watt bulb lit for 6 hours daily per person:

$$\text{Lighting} = 60W\left[\frac{6\ hr}{day}\right]\left[\frac{365\ days}{yr}\right]\left[\frac{1\ kW}{10^3\ W}\right]$$
$$= 131.4\ \ kWh/yr$$
$$= 100\ \ kWh/yr$$

Annual lighting needs require about 100 kWh per bulb. Other basic energy services might include refrigeration, computing, and charging of other electronics. Refrigerators are most likely used by households, so the energy requirements can be divided among all household members. Electricity may not be available (or affordable) for all-day use, but we will assume that it is in this scenario.

$$\text{Refrigeration} = 50W\left[\frac{24\ hr}{day}\right]\left[\frac{365\ days}{yr}\right]\left[\frac{1\ kW}{10^3\ W}\right]\left[\frac{refrigerator}{5\ people}\right]$$
$$= 87.6\ \ kWh/yr$$
$$= 90\ \ kWh/yr$$

$$\text{Computing \& other electronics} = 100W\left[\frac{2\ hr}{day}\right]\left[\frac{250\ days}{yr}\right]\left[\frac{1\ kW}{10^3\ W}\right]$$
$$= 50\ \ kWh/yr$$

These calculations provide the basis for a basic energy budget:

$$\text{Basic energy needs (low)} = (\text{heating \& cooking}) + (\text{lighting}) + (\text{refrigerator \& computing})$$
$$= \left(300\,\frac{kWh}{yr}\right) + \left(100\,\frac{kWh}{yr}\right) + \left(90\,\frac{kWh}{yr} + 50\,\frac{kWh}{yr}\right)$$
$$= 540\ \ kWh/yr$$
$$\text{Basic energy needs (high)} = \left(2{,}000\,\frac{kWh}{yr}\right) + \left(100\,\frac{kWh}{yr}\right) + \left(90\,\frac{kWh}{yr} + 50\,\frac{kWh}{yr}\right)$$
$$= 2{,}240\ \ kWh/yr$$

Based on these calculations, the most basic of energy budgets requires between about 540 and 2,240 kilowatt-hours per person each year, including about 300 kilowatt-hours of electricity.

Example 10 Goldemberg (1996) identifies 1 tonne of oil equivalent (toe) as the amount of primary energy that corresponds with improvements on several development metrics. About how much delivered energy is this in kilowatt-hours?

The unit *tonne of oil equivalent* is defined by the International Energy Association (IEA) as:

$$1\ toe \equiv 41.868\ GJ$$

With this information, the problem becomes a straightforward conversion calculation:

$$1\ toe/year = 41.868\ \frac{GJ_{th}}{year}\left[\frac{10^9 J_{th}}{1\ GJ_{th}}\right]\left[\frac{0.33\ J_e}{1\ J_{th}}\right]\left[\frac{1kWh}{3.6\times10^6 J_e}\right]$$

$$= 3{,}837.9\ \ kWh/year$$

$$= 3{,}800\ \ kWh/year$$

This finding suggests quite a gap between what a minimal energy budget can look like and the levels of per capita energy consumption associated with improvements on development indicators, such as decreased illiteracy, reduced infant mortality, and increased life expectancy.

5. LIGHT BULBS AND OIL

This section has been adapted from Purcell, Edward, "Energy in a Light Bulb and in Oil." *American Journal of Physics.* Available at http://ajp.dickinson.edu/Readers/backEnv.html. Updated 31 May 2012.

How much oil do we use to obtain basic services? Let's start by looking at the energy equivalences of services and supplies.

Example 11 How much energy, and how many barrels of oil, are needed to keep a conventional incandescent 60-watt light bulb lit continuously for one year?

$$\text{Energy to light a 60-watt bulb} = 60\ watts \times 1\ year$$

$$= 60\ W{\cdot}yr \left[\frac{J}{W \cdot s}\right]\left[\frac{60\ sec}{min}\right]\left[\frac{60 min}{hr}\right]\left[\frac{24 hr}{day}\right]\left[\frac{365 days}{yr}\right]$$

$$= 60\ J_e$$

$$= 1.89\times10^9\ J_e$$

$$= 2\times10^9 J_e$$

Example 12 How many barrels of oil are needed to keep a conventional incandescent 60-watt light bulb lit continuously for one year

To answer this question, we begin by calculating how much energy one barrel (bbl) of oil contains. Doing this requires two additional pieces of information: (1) the energy content of one gram of oil, which is about 10^4 calories per gram, and (2) the density of petroleum, which ranges from about 800 kg/m^3 for light sweet crude to 1,000 kg/m^3 for heavy crude oil. Since light sweet crude is preferred, we'll assume a low density of 800 kg/m^3.

$$\text{Energy in 1 bbl of oil} = 1bbl\left[\frac{42\ gal}{bbl}\right]\left[\frac{3.79\ l}{gal}\right]\left[\frac{m^3}{10^3\ l}\right]\left[\frac{800\ kg}{m^3}\right]\left[\frac{10^3 g}{kg}\right]\left[\frac{10^4 cal}{g}\right]\left[\frac{4.1868\ J}{cal}\right]$$

$$= 5.33\times10^9\ J_{th}/bbl$$

$$= 5\times10^9\ \ J_{th}/bbl$$

Of this energy, a conventional power plant can deliver about 30% of the energy content as usable energy. Assuming an efficiency factor (η) of 0.3, how much usable energy can one barrel of oil deliver to light a light bulb?

$$\text{Electricity output from 1 barrel of oil} = (\text{Energy content of fuel})\times(\text{Power plant efficiency})$$

$$= 5.33\times10^9 J_{th} \times 0.3$$

$$= 1.60\times10^9 J_e$$

$$= 2\times10^9 J_e$$

So keeping one 60-watt light bulb on all day, every day for an entire year consumes about the energy equivalent of an entire barrel of oil combusted in a typical power plant.

Importantly, although this problem compares the amount of energy used by a light bulb and the amount of energy content in a barrel of oil, it should not be understood to say that the light bulb is being powered by oil. Instead, the purpose of this problem is to help provide a better sense of the energy content of a common fuel and the energy consumed by a common technology. In the United States, very little electricity is generated from petroleum products.

6. Daily Energy from the Sun

This section has been adapted from Purcell, Edward, "Daily Energy from the Sun." *American Journal of Physics.* Available at http://ajp.dickinson.edu/Readers/backEnv.html. Updated 31 May 2012.

Let's define a "sun day" as the amount of energy received by the whole Earth in one day from the sun. As a figure of merit, the entire global coal reserves have been estimated at 10 sun days. How many cubic miles of coal does that amount to? How does it compare with the amount of carbon in the Earth's atmosphere, of which about 1 molecule in 3,000 today is CO_2?

As always, begin with the two key issues in simplifying such a statement:

1. What are the units?
2. What are the constants you know?

Let's begin by calculating the amount of energy received by the Earth from the sun in a single day. For now, we will consider a simple model and will develop a more complex model, including atmospheric interactions, in Chapter 10 when we look at sustainability.

Example 13 How much energy is in a single "sun day"? Give your answer in Joules.

The power density of sunlight, the amount of sunlight per unit of area, outside of the atmosphere is about 1400 W/m^2. The area of sunlight intercepted by the Earth is equal to a circular disk of the Earth's radius, which is about 6.38×10^6 m. With these values we can calculate the amount of energy in a single "sun day."

$$\begin{aligned}\text{One "sun day"} &= (\text{Area of intercepted sunlight})\times(\text{Power density of sunlight})\times(\text{Time})\\ &= [\pi R^2]\left[1400\frac{W}{m^2}\right][1day]\left[\frac{24hr}{1day}\right]\left[\frac{3600seconds}{1hr}\right]\\ &= [\pi\cdot(6.38\times10^6m)^2]\left[1.2\times10^8\frac{W\cdot s}{m^2}\right]\\ &= 1.5\times10^{22}J\end{aligned}$$

Example 14 If global coal reserves have as much energy as 10 sun days, what is the mass of global coal reserves?

Now we can calculate the mass and volume of coal equal to 10 sun days using the energy content of coal, about 2.93×10^7 J/kg, and the density of coal, about 1.24 g/cm^3 for bituminous coal.

$$\begin{aligned}\text{Global coal reserves by mass} &= (\text{10 sun days})\times\left(\text{Coal}_{\text{energy content}}\right)\\ &= [10][1.5\times10^{22}J]\left[\frac{1kg}{2.93\times10^7J}\right]\\ &= 5.112\times10^{15}kg(coal)\\ &= 5\times10^{15}kg(coal)\end{aligned}$$

Example 15 If global coal reserves have as much energy as 10 sun days, what is the volume of global coal reserves?

$$\text{Global coal reserves by volume} = (\text{Global coal reserves by mass}) \times (\text{Coal}_{\text{density}})$$
$$= [5.112 \times 10^{15} kg] \left[\frac{10^3 g}{1 kg}\right] \left[\frac{1 cm^3}{1.24 g}\right] \left[\frac{1 m^3}{10^6 cm^4}\right]$$
$$= 4.128 \times 10^{12} m^3 (coal)$$
$$= 4 \times 10^{12} m^3 (coal)$$

Example 16 If one molecule in 3,000 is CO_2, what is the mass of carbon present in the atmosphere in the form of CO_2?

The set-up for this problem requires knowing the mass of the Earth's atmosphere is about 5×10^{18} kg and that the average molar mass of air is about 28.96 g/mol. With this, we can easily calculate the mass of carbon present in the atmosphere in the form of CO_2.

$$\text{Mass of C} = \left(\frac{\text{Mass of atmosphere}}{\text{Average molar mass of air}}\right) \times \left(\frac{1 \text{ mole } CO_2}{3000 \text{ mole air}}\right) \times \left(\frac{\text{Mass of C}}{\text{Mole of } CO_2}\right)$$
$$= \left[\frac{5 \times 10^{21} g(air)}{28.96\, g(air)/mol(air)}\right] \left[\frac{1 mol(CO_2)}{3000 mol(air)}\right] \left[\frac{12 g(C)}{1 mol(CO_2)}\right] \left[\frac{1 kg}{10^3 g}\right]$$
$$= 7 \times 10^{14} kg$$

Globally then, there is roughly ten times as much carbon in recoverable coal as in atmospheric CO_2. The world petroleum reserve, incidentally, is more like *one* sun day. What this also means, very simply, is that *if* we insist on burning all these fossil fuels, the world will run out of atmosphere in which to put the waste long before we run out of this resource. Going forward, one can therefore already see that until other uses for coal are discovered that do not involve burning it, coal has been a great resource for humanity, and if we can't find a way to extract its energy economically without all the associated emissions, it remains a great resource…. to prop up the ground.

This problem is both a great first comparison of fossil and renewable energy resources, but is also adapted from the work of Edward Purcell, who ran a monthly column in the *American Journal of Physics* where the back-of-the-envelope calculation became a true art form. Ed Purcell worked during WWII to develop radar at the famous 'rad lab' at MIT, and he won the Nobel Prize at the age of 40 for work on nuclear magnetic resonance. As a graduate student at Harvard, I spent a number of late afternoons walking to his small office on the top floor of Jefferson Laboratory, where Ed said the 'extinct' professors were housed in the twisting short corridors around the library.

The surest way to launch a fascinating – and often challenging – conversation with Ed was to pose or to ask him about a back-of-the-envelope calculation. At that time, my interests were most closely connected to cosmology, and the weight of the universe, the speed of light on the lip of a black hole, and on occasion the energy from the sun, were great ways to get Ed to reveal how much the back of the envelope was, in fact, his preferred means to explore the universe.

7. HUMBLE OIL: THE POWER TO MELT GLACIERS

ExxonMobil traces its roots to the Humble Oil Company, in the town of Humble, which was chartered in Texas in February 1911 with an initial investment of $150,000 in capital (raised to $300,000 in 1912). In 1917 Humble had 217 wells and a daily crude oil production of about 9,000 barrels.

By the 1960s Humble had both grown, and provided, unintentionally, an incredible opportunity to apply back-of-the-envelope assessment tools to evaluate their environmental impact! Before examining the claim they made in *Life Magazine* (below), take a look at the production figures from Humble as a way to get comfortable with some energy units and unit conversions:

Example 17 How many gallons of crude oil was Humble producing daily in 1917?

$$\text{Gallons of crude produced daily} = 9{,}000\,\frac{bbls}{day}\left[\frac{42gal}{bbls}\right]$$

$$= 378{,}000\,\frac{gal}{day}$$

$$= 4\times10^{5}\,\frac{gal}{day}$$

Example 18 Assume that about 45% of crude oil is converted to gasoline, how many gallons of gasoline could be produced daily (US DOE, 2011b)?

Note: since this question builds on the result from Example 17, we begin with 378,000 gal/day to carry as many significant figures as possible and get the most accurate answer.

$$\text{Gallons of gasoline produced daily} = 378{,}000\,\frac{gal(oil)}{day}\times0.45$$

$$= 170{,}100\,\frac{gal(gas)}{day}$$

$$= 2\times10^{5}\,\frac{gal(gas)}{day}$$

Example 19 If the average person drives 30 miles per day in a vehicle that gets 27.5 miles per gallon, how many drivers can this level of production support?

$$\text{\# of drivers} = 170{,}100\,\frac{gal(gas)}{day}\left[\frac{27.5\ miles}{gal}\right]\left[\frac{driver\text{-}day}{30\ miles}\right]$$
$$= 155{,}925\ drivers$$
$$= 2\times10^{5}\ drivers$$

As you grow more familiar with energy units and making order-of-magnitude estimates, you will start being able to do these kinds of calculations – in the margins, on the back of an envelope, or in your head. When reading articles about energy, be they in the newspaper, a magazine, or an academic journal, doing these kinds of quick calculations can be an easy way to translate unfamiliar values into more meaningful or familiar terms.

With a better sense of how much oil Humble Oil Company was producing in 1917, let's return to the bigger question at hand. Humble Oil Company's production expanded steadily over time. During World War II, Humble became the largest domestic producer of crude oil and continued in that position into the 1950s. By 1949 the company was operating 9,928 oil wells and had a net production of 275,900 barrels (43,860 m^3) daily of crude oil and 15,900 barrels (2,530 m^3) daily of natural-gas liquids. Take a moment to calculate how many modern drivers Humble's 1949 production levels could support.

In 1959 Humble and Standard Oil of New Jersey consolidated domestic operations. By the end of the year Esso Standard and the Carter Oil Company, other affiliates of Standard of New Jersey, were incorporated into Humble, and in 1960 they were joined by other affiliates including Enjay Chemical, Pate Oil, Globe Fuel Products, and Oklahoma Oil. The restructuring allowed the new Humble Company to reduce duplication and costs and to coordinate all of its domestic activities more effectively. The Humble workforce dropped by a quarter in the first five years after the merger, while its profits doubled. Humble's restructuring also allowed both companies to sell and market gasoline nationwide under the Esso, Enco and Humble brands.

In a 1962 edition of *Life Magazine*, Humble printed the following remarkable advertisement showing Alaska's Taku Glacier. In the text they made a specific assessment of their energy 'footprint' which we can now see in several different ways!

Figure 5 Advertisement for Humble Oil in *Life Magazine,* 1962

Source: *Life Magazine*, 1962

The text on the advertisement reads:

> Each day Humble supplies enough *energy* to melt 7 million tons of glacier!
>
> This giant glacier has remained unmelted for centuries. Yet, the petroleum energy Humble supplies—if converted into heat—could melt it at the rate of 80 tons each second! To meet the nation's growing needs for energy, Humble has applied science to nature's resources to become America's Leading Energy Company. Working wonders with oil through research, Humble provides energy in many forms—to help heat our homes, power our transportation, and to furnish industry with a great variety of versatile chemicals. Stop at a Humble station for new Enco Extra gasoline, and see why the "Happy Motoring" Sign is the World's First Choice!

Example 20 How much crude oil must Humble produce daily in order to melt 7 million tons of glacier? Assume that all of the ice is right at the freezing point.

$$\text{Crude Oil produced daily} = 7\times10^{6} tons \left[\frac{1\ tonne}{1.102\ tons}\right]\left[\frac{10^{3} kg}{1\ tonne}\right]\left[\frac{3.33\times10^{5} J}{1\ kg}\right]$$

$$= 2.12\times10^{15} J$$

$$= 2\times10^{15} J$$

Recall from Example 11, that there are about 5×10^9 joules per barrel of oil to calculate the volume of crude produced daily:

$$\text{Crude oil produced daily} = 2.12\times10^{15} J \left[\frac{1\ barrel}{5.33\times10^{9} J}\right]$$
$$= 3.97\times10^{5}\ barrels$$
$$= 4\times10^{5} barrels$$

8. Understanding Impact: The IPAT Relation

A particular 'equation' that has been analytically annoying but computationally incredibly useful is the unit identity known as the "IPAT" relation. There is no better way to start a fight among energy and sustainability researchers than to delve into the history of IPAT, and some pretty harsh names have been called with one researcher demeaning another in this fracas. For a sampling of the heated debate, the war of calculations and of words can be seen in the work of the different sides with Ehrlich and Holdren (1971) and Commoner, (1972) two of the more academic and at times less ideologically fervent entries on this issue.

8.1. Introducing IPAT

If we first stick to the formalism, not the fracas, the IPAT is the 'relation', or 'identity', or 'equation', depending who you are, with the form given in Equation 12:

$$\text{Impact} = (\text{Population}) \times (\text{Affluence}) \times (\text{Technology}) \tag{12}$$

The IPAT was one of the earliest attempts to describe the role of multiple factors in determining environmental degradation. It describes the multiplicative contribution of population, affluence, and technology to environmental impact. Let's take a closer look at each of the IPAT components:

- **I***mpact*: the magnitude of the environmental impact in question may be expressed in terms of resource depletion or waste accumulation.
- **P***opulation*: the size of the impacting human population.
- **A***ffluence*: the level of consumption by that population.
- **T***echnology*: the processes used to obtain resources and transform them into useful goods and wastes.

Note that – central to the debate over this 'concept' – there is no mathematical reason for these quantities, although they do seem very reasonable.

The formula was originally used to emphasize the contribution of a growing global population on the environment, at a time when world population was roughly half of what it is now. It continues to be used with reference to population policy.

The IPAT relation can be applied conceptually to help characterize the interplay of different dynamics on environmental impact. For example, consider the case of developing countries where affluence should increase and population may increase. The IPAT relation provides a conceptual framework that suggests that if technology remains the same, environmental impact will increase, but also that improved technology could reduce the level of environmental impact associated with increases in affluence and/or population.

8.1.1. Operationalizing IPAT Mathematically

The IPAT relation can also be operationalized mathematically through the association of data with each of the factors in the identity. Indeed, one interesting aspect of the IPAT worth highlighting is that an "IPAT" relationship can be constructed out of any reasonable set of quantities whose units can be described in a way that the identity form "x = x" can be maintained. In fact, just as back-of-the-envelope calculations can be built around unit analysis assessments, an infinite number of different IPAT identities can be constructed. For example, Equation 13 provides an IPAT formulation appropriate to the energy context, while Equation 14 shows how this formulation might be operationalized.

$$\text{Energy use} = (\text{Population})\times(\text{Affluence})\times(\text{Energy Intensity}) \tag{13}$$

$$\text{Energy use [J]}=(\text{Population [\# of persons]}) \times \left(\text{Affluence} \left[\frac{\$\text{GDP}}{\text{person}}\right]\right) \times \left(\text{Energy } \left[\frac{\text{J}}{\$\text{GDP}}\right]\right) \tag{14}$$

You can really go to town in constructing version after version of these simple but useful identities. While the examples are endless, a number of very useful computational forms appear. IPAT equations can be used to represent technologies (*e.g.*, light-duty vehicles) and sectors (*e.g.*, electric power), as well as a wide range of whole systems. While none of these are particularly satisfying as a 'theory of everything, energy', they do begin to bring the full system into perspective.

In an important re-assessment of IPAT, Paul Waggoner and Jesse Ausubel (2002) built a nice typology of IPAT formulations. Table 8 summarizes their categorization of the words and symbols representing the forces represented in IPAT and its variants, the actors that influence these forces, and the dimensions or units used to operationalize them for energy emissions, see Table 8.

Table 8 Typology of IPAT variations

Category	Symbol	Actors / Drivers	Dimension for energy emissions
Impact	I	All	Emissions
Population	P	Parents/policies	Capita
Affluence	A	Workers	GDP/Capita
Intensity of use	C	Consumers	Energy/GDP
Efficiency	T	Producers & users	Emissions/Energy
Consumption/capita	A × C		Energy/Capita
Consumer challenge	P × A × T		GDP × (Emissions/Energy)
Technology challenge	P × A × C		Energy
Sustainability challenge	P × A		GDP
Sustainability levers	C × T		Emission/GDP

Source: Waggoner and Ausubel, 2002.

Once an IPAT model has been constructed, data can be applied both to calculate the impacts of an energy system and to evaluate the effects of various changes brought about by shifts in technology or policy.

8.1.2. Assuming Exponential Growth

The field of industrial ecology, in particular, has applied a technique called *decomposition analysis* to IPAT formulations. Decomposition analysis relies on the assumption that all of the factors employed – population, affluence, and technology – grow exponentially. This assumption allows each of the factors to be re-written in exponential form:

$$\text{Population:} \quad P_t = P_0 e^{r_{population} t} \tag{15}$$

$$\text{Affluence:} \quad A_t = A_0 e^{r_{affluence} t} \tag{16}$$

$$\text{Technology:} \quad T_t = T_0 e^{r_{technology} t} \tag{17}$$

Impact can then be re-written as a function of these three exponential growth functions, as shown in Equations 18.

$$\text{Impact}_t(\text{P}_t, \text{A}_t, \text{T}_t) = (P_0 e^{r_{population} t})(A_0 e^{r_{affluence} t})(T_0 e^{r_{technology} t}) \quad (18)$$

Note that the assumption of exponential growth for all of the factors on the right side of the equation means that impact will also exhibit exponential growth and the rate at which the impact grows will be a function of the growth rates of the various factors, Equation 19.

$$r_{emissions} = r_{population} + r_{affluence} + r_{technology} \quad (19)$$

With decomposition analysis, the change in impact between time periods is broken down, or decomposed, into the individual factors. In essence, it provides a method for quantifying the portion of the impact that can be attributed to a particular factor over a given period of time or how changes in the factors might influence the level of impact in the future. The following sections provide examples of how IPAT can be used to analyze greenhouse gas emissions.

8.2. Applying IPAT to Global Greenhouse Gas Emissions from Energy Use

Although any number of IPAT identities might be constructed to think about greenhouse gas emissions, let's begin by focusing on the impact of carbon dioxide (CO_2) emissions from global energy use.

The conceptual structure of such an identity is given in Equation 20, wherein the single *technology* factor is divided into two complementary terms, *energy intensity* and *carbon intensity*:

$$\text{CO}_2 \text{ emissions} = (\text{Population})\times(\text{Affluence})\times(\text{Energy Intensity})\times(\text{Carbon Intensity}) \quad (20)$$

In this case, *impact* refers to annual CO_2 emissions from global energy use, *population* refers to the global population, and *affluence* might reasonably be thought of as per capita gross domestic product. The importance of our two technology terms becomes clear as we think about how to operationalize our conceptual identity. *Energy intensity* and *carbon intensity* need to relate CO_2 emissions to GDP, which is easily accomplished by these two terms which respectively represent the amount of energy consumed per dollar of GDP produced and the amount of CO_2 emitted per unit of energy consumed. Put all of this together, and we see how our conceptual identity can be operationalized as given in Equation 21:

$$\text{CO}_2\left[\frac{\text{t(CO}_2)}{\text{yr}}\right] = (\text{Population})\times\left(\text{Affluence}\left[\frac{\$\text{ GDP}}{\text{person}\cdot\text{yr}}\right]\right)\times\left(\frac{\text{Energy}}{\text{Affluence}}\left[\frac{\text{MJ}}{\$\text{ GDP}}\right]\right)\times\left(\frac{\text{Carbon}}{\text{Energy}}\left[\frac{\text{t(CO}_2)}{\text{MJ}}\right]\right) \quad (21)$$

Using this identity and the exponential decomposition highlighted above, we can use empirical data about changes in the *population*, *affluence*, *energy intensity*, and *carbon intensity* terms to forecast how CO_2 emissions might change over time. That is, we can use the growth rates of the terms on the right side of the equation to calculate a growth rate for the left-hand term, CO_2 emissions.

Example 21 Based on the data given in Table 9, what is the annual growth rate for CO_2 emissions from global energy use?

Table 9 Annual growth rates for population, affluence, energy intensity, and carbon intensity

Population	Affluence	Energy Intensity	Carbon Intensity
1.40%/yr	1.53%/yr	-0.97%/yr	-0.24%/yr

To solve this problem, we first assume that *population*, *affluence*, *energy intensity*, and *carbon intensity* are growing exponentially. This assumption allows us to use the decomposition analysis introduced in the previous section.

From Equation 19, we know that summing the growth rates on the right side of the equation will give us the growth rate for the left side of the equation, in this case the annual growth rate of CO_2 emissions from energy use:

$$\begin{aligned} r_{CO_2\ emissions} &= r_{population} + r_{affluence} + r_{energy\ intensity} + r_{carbon\ intensity} \\ &= 1.4\,\%/yr + 1.53\,\%/yr + (-0.97\,\%/yr) + (-0.24\,\%/yr) \\ &= 1.72\,\%/yr \end{aligned}$$

$$I_t = I_0 e^{0.00172t}$$

Example 22 If global CO_2 emissions from energy equaled 30 gigatons in 2008, what will global CO_2 emissions be in 2020?

The set-up for this problem is now quite straightforward. To forecast global CO_2 emissions in the year 2020, simply input the growth rate calculated in Example 21 and the specified dates into an exponential growth equation:

$$\begin{aligned} I_{\mathrm{t}} &= I_0 e^{r_{emissions} t} \\ \mathrm{I}_{2020} &= I_{2008} e^{(0.0172yr^{-1})(12yr)} \\ &= 30\ Gt(CO_2)\left[e^{(0.0172yr^{-1})(12yr)}\right] \\ &= 37\ Gt(CO_2) \end{aligned}$$

8.3. Applying IPAT to Greenhouse Gas Emissions from Transportation

This section has been adapted from Sager, Jalel, *et al.* (2011) "Reduce growth rate of light-duty vehicle travel to meet 2050 global climate goals," *Environmental Research Letters* **6** 024018. *References have been removed for readability; see original article for citations and additional information.*

The transportation sector, which includes light-duty vehicles, heavy-duty trucks, buses, aviation, rail, marine, agricultural, and off-road vehicles, accounts for about 15% of global greenhouse gas emissions in terms of carbon dioxide equivalents (CO_2-e). In some transportation-heavy economies, like California, transportation already accounts for a much larger share of total annual emissions, closer to 40%. Light-duty vehicles – a category that includes most personal vehicles like cars, small trucks, and SUVs – currently account for about 45% of this sector's emissions, or roughly 6% of global emissions. Emissions from light-duty vehicles are also increasing rapidly in many emerging economies.

International agreements have advocated limiting temperature increase to 2°C or less, which limits the total amount of greenhouse gas emissions that can be moved to the atmosphere. Under plausible assumptions and emissions pathways, year 2050 global carbon dioxide (CO_2) emissions levels consistent with a 2°C temperature limit can be close to 80% below year 2007 emissions.

Even more dramatic reductions may be sought from light-duty vehicles because of the relatively large number of available mitigation options. The challenge of reducing transportation emissions is often framed as a technological challenge. Low-carbon fuel standards aim to stimulate production of fuels that produce fewer GHGs per unit energy; vehicle efficiency policies aim to reduce the fuel used and emissions produced per distance traveled. However, the benefit of using of these advanced technologies is dependent upon the level of adoption of these technologies and the associated reduction in energy use and carbon intensity. A third option is also available: reduce emissions by reducing the number of vehicle-miles traveled. These three primary means for reducing transportation emissions can be decomposed into an IPAT identity, Equation 22:

$$\mathrm{CO}_{2,\,\mathrm{transport}} = (\mathrm{Population}) \times (\mathrm{Transport\ Intensity}) \times (\mathrm{Energy\ Intensity}) \times (\mathrm{Carbon\ Intensity}) \quad (22)$$

Recall that an important feature of an analysis using an IPAT formulation is that data sets are available for each quantity individually. Once units for each term have been clearly defined, the next step involves checking that suitable data sets, or simple analytic expressions, are available for each term. In the case of our transportation IPAT, we can conduct an analysis with the following units:

$$\mathrm{CO}_{2,\,\mathrm{transport}} = (\mathrm{Population}) \times \left(\frac{\mathrm{Transport}}{\mathrm{Person}}\left[\frac{\mathrm{miles}}{\mathrm{person}}\right]\right) \times \left(\frac{\mathrm{Energy}}{\mathrm{Transport}}\left[\frac{\mathrm{MJ}}{\mathrm{mile}}\right]\right) \times \left(\frac{\mathrm{Carbon}}{\mathrm{Energy}}\left[\frac{\mathrm{gCO_2}}{\mathrm{MJ}}\right]\right) \quad (23)$$

With these units established, our next move is to find or derive data for each term.

Equations 22 and 23 can be put to use in exploring where changes in the carbon footprint of transportation are possible, and what level of changes are needed to secure our future. The importance of altering the impact of transportation embodies many of the features that IPAT equations are well suited to address. To see how, let's consider how IPAT can be used to analyze the potential emissions reductions from light-duty vehicles required for achieving 2050 climate goals.

Specifically, let's consider how IPAT can be used to understand whether innovation in a single area, such as fuel economy, offer a realistic, affordable, or resilient pathway to the light-duty vehicle emission reductions necessary by mid-century.

First, consider the greenhouse gas mitigation challenge posed by light-duty vehicles. In 2050, assuming a global population of 9 billion, achieving the 80% emissions reductions implies per capita annual light-duty vehicle emissions of only 50 to 100 kg of carbon dioxide equivalent (CO_2-eq). A survey of light-duty vehicle usage and fuel economy in an economically diverse set of countries finds that per capita greenhouse gas emissions from light-duty vehicles spans a wide range, from about 100 to 4,000 kg CO_2-equivalent per year. These differences are principally explained by differing national per capita light-duty-vehicle use, rather than by fleet average fuel efficiency and carbon intensity factors, which reflect broadly similar car technology worldwide. In upper-income countries, intensive light-duty vehicle use results in present-day emissions that exceed the 2050 target range of 50–100 kg CO_2-eq per year by a factor of 10–80.

How might IPAT be used to explore the policy options available to achieve the implied per capita emissions targets? To explore this question, a decomposition can be used that expresses per-capita light-duty vehicle emission in terms of two sets of variables: propulsion carbon intensity, the amount of CO_2 emitted per kilometer traveled, and per-capita vehicle use, see Equation 24.

$$\text{Per Capita CO}_2\text{ Emissions} = (\text{Propulsion Carbon Intensity})\times(\text{Per Capita Vehicle Use}) \quad (24)$$

Current policy discussion is dominated by attention to the first set of variables, which can further be decomposed into two terms representing carbon intensity (carbon per unit of energy consumed) and energy intensity (energy per vehicle-kilometer traveled or VKT), see Equation 25.

$$\text{Per Capita CO}_2\text{ Emissions} = \left[\left(\frac{\text{Carbon}}{\text{Energy}}\right)\times\left(\frac{\text{Energy}}{\text{VKT}}\right)\right]\times(\text{Per Capita Vehicle Use}) \quad (25)$$

From this further decomposition, we see that the first set of variables can be improved by decreasing the carbon intensity of fuels, *e.g.*, through a low-carbon fuel standard, or decreasing the energy intensity of travel, *e.g.,* through increasing vehicle efficiency, or by changing both.

The second set of variables representing per capita light-duty vehicle use can also be further decomposed, see Equation 26. Emissions from vehicle use depend upon three variables: vehicle occupancy, expressed here as vehicle-kilometers traveled per person kilometer traveled (PKT); the distance of the trip, expressed here as person-kilometer traveled per trip; and the annual number of per capita trips.

$$\text{Per Capita CO}_2\text{ Emissions} = \left[\left(\frac{\text{Carbon}}{\text{Energy}}\right)\times\left(\frac{\text{Energy}}{\text{VKT}}\right)\right]\times\left[\left(\frac{\text{VKT}}{\text{PKT}}\right)\times\left(\frac{\text{PKT}}{\text{Trip}}\right)\times\left(\frac{\text{Trips}}{\text{Person-year}}\right)\right] \quad (26)$$

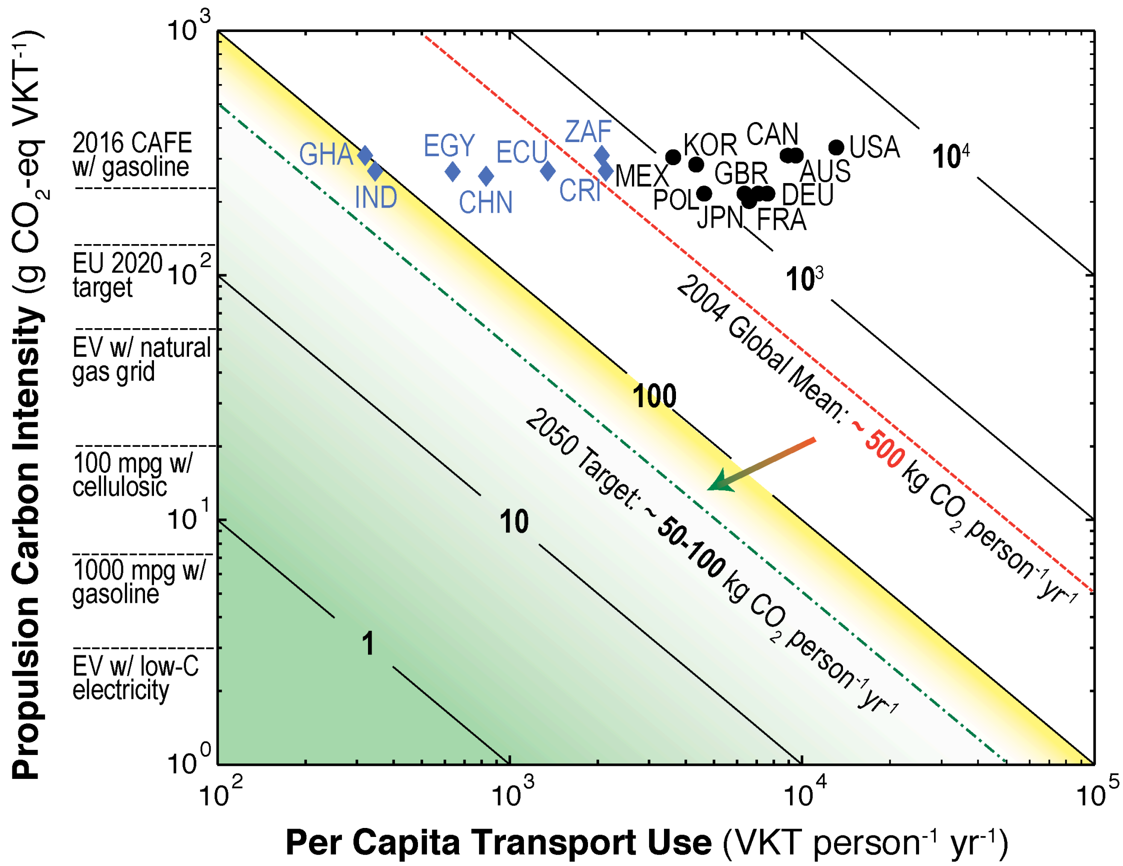
This further decomposition of the second set of variables suggests three broad strategies for improvement: increasing vehicle occupancy rate, decreasing the mean per-trip distance, and reducing the per-capita trip rate.

With this decomposition in mind, let us now return to our dataset of 2007 per capita emissions from various countries. Figure 6 maps current per capita emissions onto a graph that shows how these two sets of variables interact to achieve the 2050 per capita targets. Propulsion carbon intensity is shown on the vertical axis, with text highlighting indicative vehicle standards and technologies, while per-capita light-duty vehicle use is shown on the horizontal axis.

Any given per-capita transportation CO_2 target can be met through combinations of policies to reduce propulsion carbon intensity and/or vehicle usage (see the diagonal iso-lines). Ambitious transport CO_2 targets (such as the yellow 2050 target region, roughly consistent with established 2°C climate goals) require widespread deployment of extremely efficient vehicles and low-carbon fuels.

Moreover, even with a dramatic (~10x) reduction in propulsion carbon intensity, meeting such targets requires transport use substantially lower (≥2x) than currently typical of most industrialized countries. For example, with global per-capita light-duty vehicle use of 10,000 km per year (U.S. per capita use is closer to 20,000 km per year), carbon propulsion intensity would need to decline from current levels of ~300 to ~5-10 g CO_2-eq per km on a "well-to-wheel" (WTW, fuel lifecycle) basis.

Figure 6 GHG Emissions from Light-duty Vehicle Use: Nations and Technologies

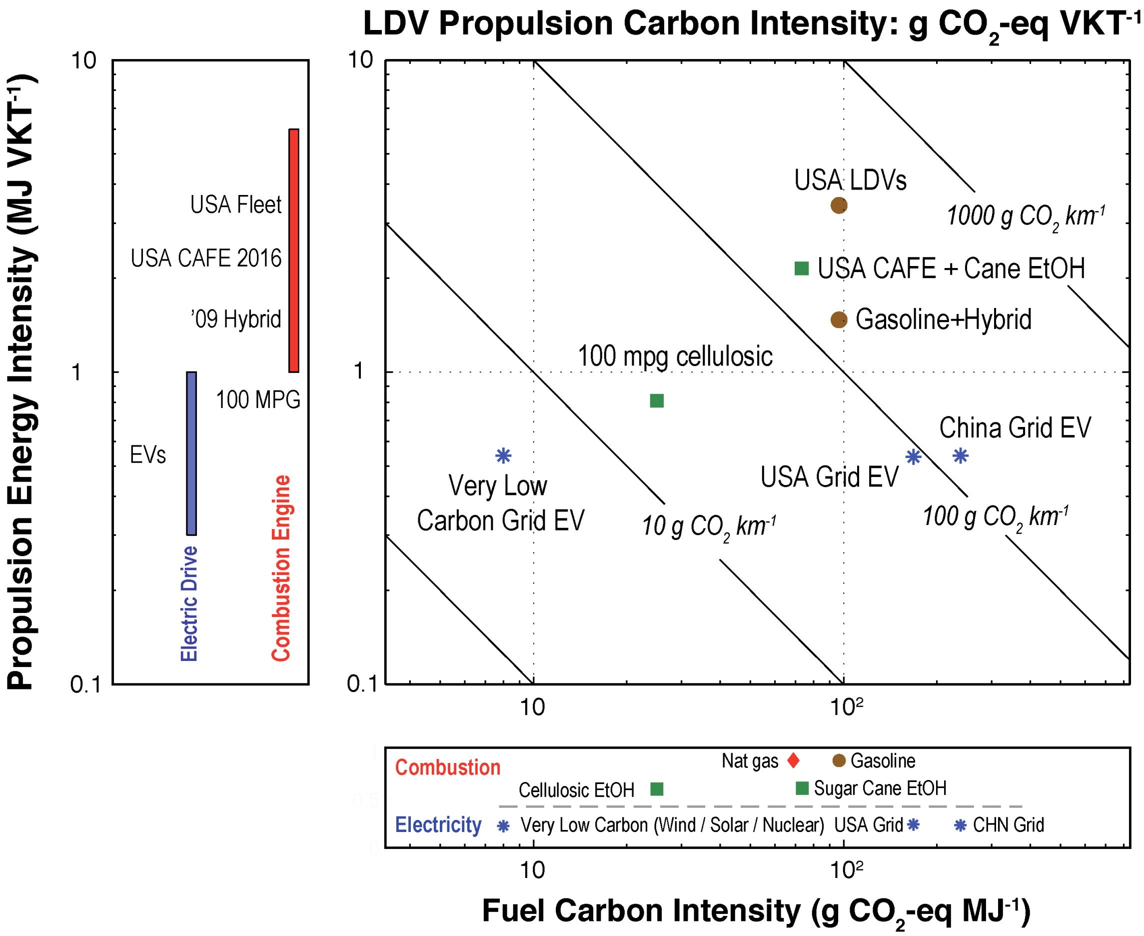


Notes: Average per-capita light-duty vehicle transport CO_2 emissions (kg CO_2 person^{-1} yr^{-1}) for a global sample of countries (2007) with a wide range of per-capita incomes. Blue diamonds denote low-income nations, and black circles are medium/high-income OECD member states.

The matrix of vehicle technology options in Figure 7 shows that this performance level would require universal deployment of one or more of the following clusters: electric vehicles (EVs) running on nearly zero-carbon electricity, cellulosic biofuel-powered vehicles achieving 300 miles per gallon (mpg), or gasoline-fueled vehicles achieving in excess of 1,000 mpg. The 1,000 mpg gasoline mark illustrates an extreme 'technology solution' and indicates the exceptional demand that would be placed on propulsion technology in order to meet climate targets if 2050 global transport use were to converge to current high-income country levels. Such levels of performance exceed optimistic technology scenarios for the year 2050. As a consequence, global growth of per-capita light-duty vehicle use to levels on par with those seen today in high-income countries would likely be incompatible with climate goals.

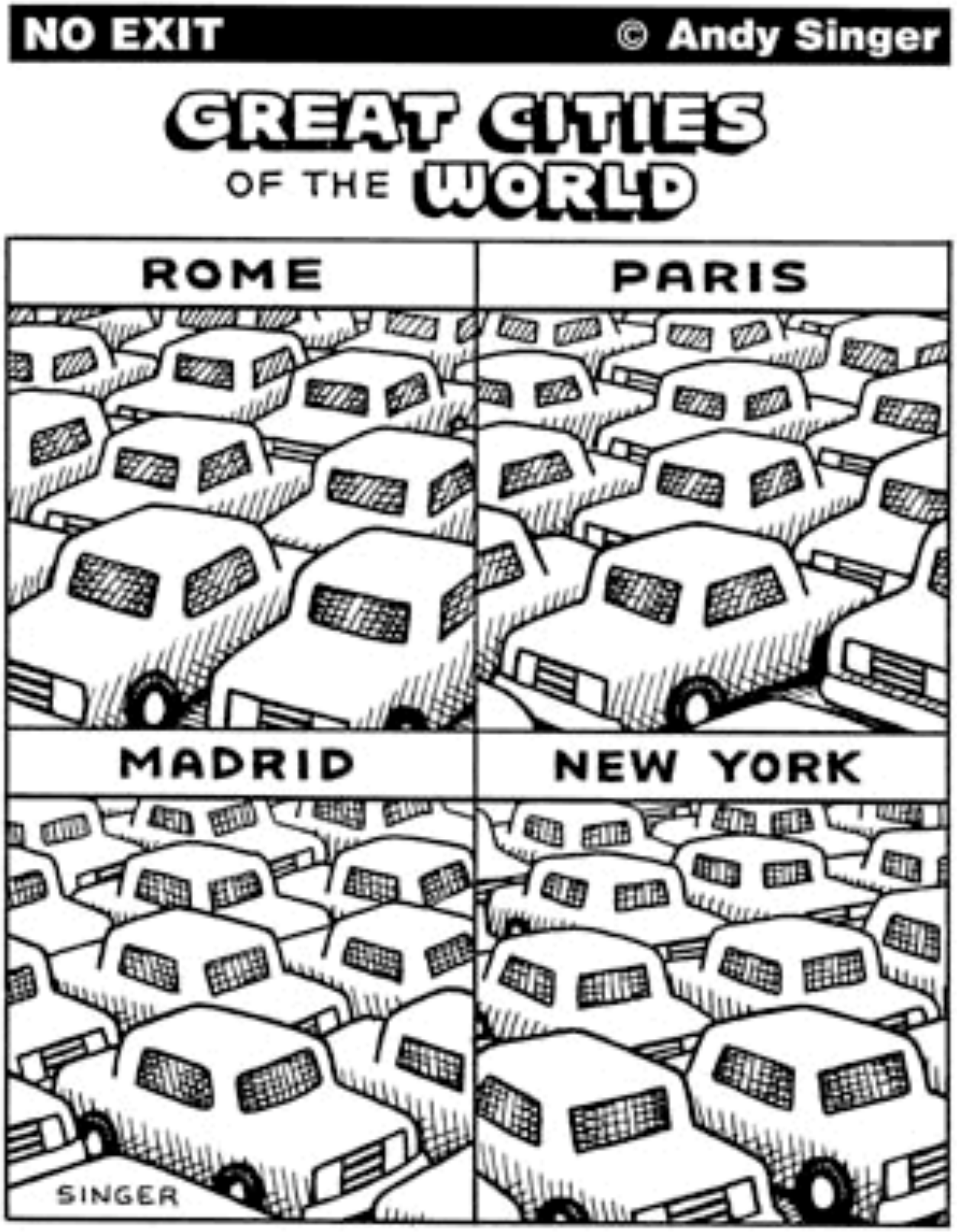
From this work, it can be seen that IPAT models are useful in allowing us to "trade off" variable values in order to define the mixes that allow us to meet the energy, social, or sustainability goals we set.

Figure 7 Propulsion Carbon Intensity Decomposed into Fuel Types and Propulsion Technology



Notes: Iso-lines indicate combinations of fuel and propulsion technologies with equal well-to-wheel (WTW) greenhouse gas emissions for light-duty vehicles. Few commercially available vehicle systems (fuel + vehicle) currently provide well-to-wheel mobility at less than 100 g CO_2 per km. In the medium term, combinations of low-carbon biofuels, clean electricity, and efficient electric or plug-in hybrid electric vehicles may offer WTW performance substantially below 100 g CO_2 per km. In the long term, dramatically lower light-duty vehicle emissions may be possible given a sufficiently large supply of near-zero CO_2 electricity.

Figure 8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